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Spin-Hall transport in a two-dimensional electron system with magnetic impurities

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Abstract. The spin Hall transport properties in a two-dimensional electron system with both Rashba spin-orbit coupling (SOC) and magnetic impurities are investigated. Electrons are scattered by impurities through an exchange interaction that leads to spin flip-flop processes and so changes the spin Hall effect induced by the SOC. The spin Hall conductance is calculated in a 4-terminal system using the Landauer-Buttiker formula and Green function approach. In comparison with the simulation results on nonmagnetic impurities doping systems, our results reveal that the spin Hall conductance is still nonzero in a system with a large density of magnetic impurities and a finite intensity of the exchange interaction between the electrons and impurities, and its sign may be altered when the doping density and interaction strength are large enough.

PACS. 72.10.-d Theory of electronic transport; scattering mechanisms – 72.15.Gd Galvanomagnetic and other magnetotransport effects – 85.75.-d Magnetoelectronics; spintronics: devices exploiting spin polarized transport or integrated magnetic fields

1 Introduction

Recently, the study of spintronics, as an active subfield in condensed matter physics, has attracted intensive attention for its potential applications in technology. Some investigations are focused on the optical and spin transport properties in spin-orbit coupling (SOC) electron systems. In two-dimensional (2D) electron systems, two types of SOC have been actively investigated [1,2]. One is the Rashba SOC, which is expressed in the effective Hamiltonian by a term of the form $\lambda(\sigma^x p_y - \sigma^y p_x)$ with $p_{x,y}$ being the x or y component of the momentum, $\sigma_{x,y}$ the Pauli matrices describing electron spin, and λ the coupling strength. The other is the Dresselhaus SOC with the form of $\beta(\sigma^x p_x - \sigma^y p_y)$. The Rashba and Dresselhaus couplings create $\mathbf{k}$-dependent effective magnetic fields $B(\mathbf{k}) \sim \lambda(p_y, -p_x, 0)$ and $B(\mathbf{k}) \sim \beta(p_x, -p_y, 0)$, respectively, which act on the electron spin and induce spin precession through the process of electron tunneling. As a result, a transverse spin Hall current can be driven in 2D systems with SOC when there is a longitudinal voltage drop. The competition between these two different SOC's provides possible experimental applications to control the direction of the spin Hall current. On the other hand, whether the universal value $\frac{e}{8\pi}$ of intrinsic spin Hall conductance is robust against disorder and other factors is also being studied actively. Analytical results considering vertex corrections have revealed the cancellation of the spin Hall effect in the presence of arbitrarily small disorder in 2D systems with Rashba SOC due to impurity scattering [3,4].

In this paper, we focus on the effect of magnetic impurities which exit in metals or semiconductors. We calculate the change of the spin Hall conductance due to the scattering of magnetic impurities in 2D systems with Rashba SOC. The results reveal that the spin Hall conductance (SHC) is still nonzero in systems with a large density of magnetic impurities, and the sign of SHC can be changed by varying the impurity density and strength of the exchange interaction between the electrons and magnetic impurities.

2 Model and formulism

The impure $L \times L$ square lattice as shown in Figure 1 is connected with four ideal leads which serve as four electron reservoirs with different chemical potentials. The spin exchange interaction between the electron spin and magnetic impurities, which are treated as classical magnetic moments throughout the simulation, is described by $J\sigma \cdot \mathbf{n}_i \delta(\mathbf{r} - \mathbf{r}_i)$, where $\mathbf{n}_i$ and $\mathbf{r}_i$ are the unit vector along the moment and the position of the ith impurity, σ are the Pauli matrices of the electron spin, $\mathbf{r}$ is the position of the electron, and J is the coupling strength including the value of the impurity moments. The disorder is introduced by the random positions and orientations of impurities $\mathbf{r}_i$ and $\mathbf{n}_i$. Using the tight-binding representation, the

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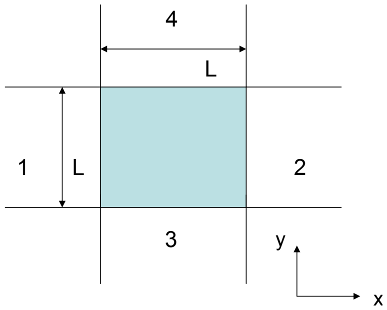

Fig. 1. A square two-dimensional sample with four ideal leads. The spin-orbit coupling and the exchange interaction between the electrons and magnetic impurities exist only in the shaded square area.

Hamiltonian of a 2D electron system with Rashba SOC and magnetic impurities has the following form

$$H = -t \sum_{\langle ij \rangle \sigma} c_{i,\sigma}^{\dagger} c_{j,\sigma} + V_{so} \sum_{i} [(c_{i,\uparrow}^{\dagger} c_{i+\delta_x,\downarrow} - c_{i,\downarrow}^{\dagger} c_{i+\delta_x,\uparrow}) - i(c_{i,\uparrow}^{\dagger} c_{i+\delta_y,\downarrow} + c_{i,\downarrow}^{\dagger} c_{i+\delta_y,\uparrow}) + \mathrm{H.c.}] + J \sum_{i,\sigma,\sigma'} c_{i,\sigma}^{\dagger} \sigma_{\sigma,\sigma'} c_{i,\sigma'} \cdot \mathbf{n}_i, \quad (1)$$

where t is the nearest-neighbour hopping energy which we set as energy units in our numerical simulation, V_{so} is the Rashba SOC strength, J is the exchange strength between the electrons and magnetic impurities, and δ_x, δ_y are vectors corresponding to the lattice spacing along the x and y axes, respectively. The positions of the magnetic dopants are random and their density is x. The 3D unit vector $\mathbf{n} = (\sin\theta_i \cos\phi_i, \sin\theta_i \sin\phi_i, \cos\theta_i)$ stands for the orientations of the magnetic moments which are uniformly distributed in the whole unit sphere.

A charge current I is driven from lead 1 to lead 2 by a potential bias. At the same time, an outgoing spin Hall current towards lead l is given by $I_{sH}^{l} = [-\hbar/(2e)](I_{l\uparrow} - I_{l\downarrow})$ [5,6], where $\uparrow$ and $\downarrow$ indicate spins parallel and antiparallel to the z-axis. The transverse spin-Hall current along lead 3 in this system is given by $I_{sH}^{(3)} = G_{sH}(V_1 - V_2)$, where $V_1 - V_2$ is the longitudinal voltage drop, and the corresponding spin Hall conductance can be directly calculated by the use of the Landauer-Büttikker formula [5,6]

$$G_{sH} = -\frac{e}{4\pi}(T_{4\uparrow,1} - T_{4\downarrow,1}),$$

where $T_{4\sigma,1}$ is the transmission coefficient from lead 1 to spin σ channel of lead 4. With the Green function technique, the spin Hall conductance can be expressed as [6–8]

$$G_{sH} = -\frac{e}{4\pi} \mathrm{Tr}(\Gamma_4 \eta G^r \Gamma_1 G^a), \quad (2)$$

where the trance is taken over the spin index, and $\eta = \pm 1$ corresponds to spin up and spin down states, respectively. $\Gamma_l = i[\sum_l - (\sum_l)^{\dagger}]$ is the retarded electron self-energy in the sample induced by the electron hopping connected with lead l. The retarded and advanced Green functions are given by $G^r = \frac{1}{E - H_c - \sum_{l=1}^{4}}$, and $G^a = (G^r)^{\dagger}$, respectively. Here, E is the Fermi energy of the electrons, and H_c is the Hamiltonian of the square sample. The self-energy can be computed by using the transfer matrices of the leads [9,10]. In our calculation, G_{sH} is averaged over 1000 realizations of randomly distributed magnetic impurities with a given concentration.

3 Numerical results

The spin Hall conductance is plotted in Figure 2 as a function of the doping density of magnetic impurities with given values of the Rashba spin-orbit coupling V_{so} and the electron-impurity exchange interaction strength J. The error bars in all the figures throughout the simulation represent the statistical errors in the structural sampling averaging. It can be seen that in most cases the spin Hall conductance is nonzero even in the presence of a relatively large density of magnetic impurities, despite the fact that the impurities tend to suppress the magnitude of spin Hall conductance to some extent. In the case of a weak exchange interaction ($J = 0.2$), almost no suppression can be seen in the curve as the density of magnetic impurities increases. For a given value of V_{so}, the suppression of the spin Hall conductance by the doping of magnetic impurities is increased by increasing the exchange interaction J. This indicates that a large electron-impurity exchange and a large density of magnetic impurities can in general destroy the spin Hall effect by spin flip-flop scattering. When electrons move near the magnetic impurities on the lattice their spin orientations may be changed due to the interaction with impurities. The increase of the impurity density enhances the probability of spin flip-flop scattering. In general, this may lead to the decrease of the spin Hall conductance. However, we notice that with some specific values of the parameters, such as $V_{so} = 0.1$ and $J = 2.5$ in Figure 2, the sign of spin Hall conductance changes when the impurity density is increased across a specific value ($x = 0.06$ in the curve for $V_{so} = 0.1$ and $J = 2.5$ in Fig. 2). This means that in the presence of SOC, the flip-flop scattering by impurities is not symmetric for up and down electron spins although the orientations of the impurity moments are random without any preferential direction. Moreover, different signs of SHC with different V_{so} values also occur due to oscillations of the sideways spin-resolved transmission coefficients (which will be shown in Fig. 4). On the other hand, the influence of nonmagnetic impurities, which scatter electrons by interaction, in the form of $\sum_{i\sigma} \varepsilon_i c_{i\sigma}^{\dagger} c_{i\sigma} \delta(\mathbf{r} - \mathbf{r}_i)$ in the Hamiltonian, is also simulated and shown in Figure 2. Here the potentials ε_i at nonmagnetic impurities are randomly distributed between $[-w/2, w/2]$ where the parameter $w = 3.0t$, and the density of impurities is x. Consistent with other analytical and numerical results [6,11], we reproduce a vanishing spin Hall conductance as the density of the nonmagnetic impurities increases. Furthermore, there is no change of SHC sign as the density of nonmagnetic impurities changes.

In Figure 3, we show the calculated G_{sH} as a function of the strength J of the exchange interaction between the

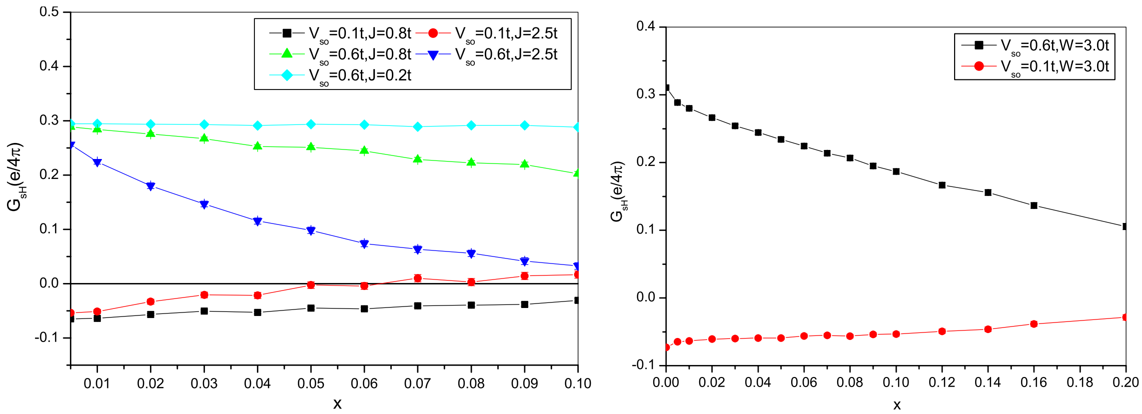

Fig. 2. Spin Hall conductance G_{sH} as a function of the density of the magnetic (left panel) and nonmagnetic (right panel) impurities for an different values of J, w and V_{so} in 20×20 square lattice for an electron energy $E = -2t$.

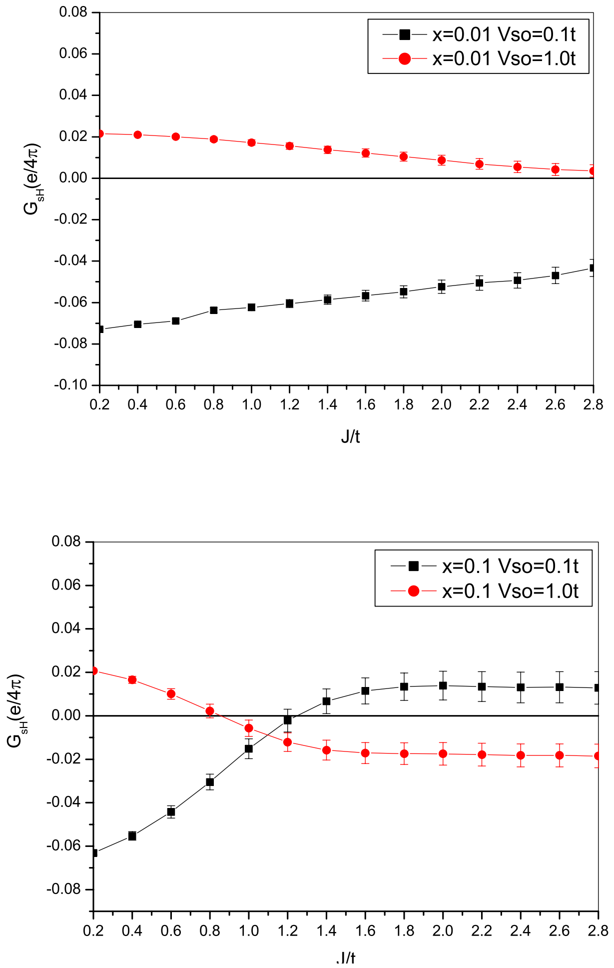

Fig. 3. Spin Hall conductance as a function of the strength of the electron-impurity exchange interaction for different values of V_{so} and x. $E = -2t$, and the system is a 20×20 square lattice.

electrons and impurities. Without any exchange interaction ($J = 0$) G_{sH} is positive in the case of $V_{SO} = 1$ but is negative in the case of $V_{SO} = 0.1$. In both cases the absolute value of G_{sH} decreases as J increases from zero due to the destructive effect of impurity scattering. This effect is stronger for a larger impurity density. Interestingly, the sign change of the spin Hall conductance as J increases can be clearly seen for a larger impurity density ($x = 0.1$) for both $V_{SO} = 0.1$ and $V_{SO} = 1$. When the exchange interaction is weak, the probability of spin flipping is small, therefore the magnitude of G_{sH} diminishes but its sign remains unchanged. As the exchange strength becomes large enough, almost all electron spins are flipped, thus the spin Hall conductance changes its sign. A larger impurity density may enhance the tendency of the sign change of G_{sH}. Combining with the results shown in Figure 2, we can see that the spin flip-flop scattering of impurities is "biased" in the presence of the SOC. This bias is usually opposite to the spin Hall conductance when there is no impurity scattering and the extent of the bias depends on the absolute value of G_{sH}.

In Figure 4, we display the SHC as a function of the strength of SOC with a nonzero density of magnetic impurities. Similar to what is found for two-dimensional systems with Anderson disorder and Rashba spin-orbit coupling, the G_{sH} curves for different values of impurity density and exchange interaction exhibit oscillations as the SOC strength is varied over the range of $0 \leq V_{so} \leq t$. Such oscillations are regarded as a manifestation of spin-resolved transmission coefficients oscillations due to the interplay of the spin precession from the SOC and the quantum interference in finite systems [5,6]. The characteristic length of spin precession is $L_{sp} \simeq \pi t/V_{so}$ [6,8], thus the oscillation period in the $G_{sH} - V_{so}$ curves can be estimated by $\delta V_{so} \simeq \pi/L$ with L being the system size. For our parameters, it is close to 0.16. The oscillation period shown in Figure 4 is consistent with this anticipated value, indicating the finite-size effect in spin procession. It can be seen that the scattering of magnetic impurities has no effect on this period, but it reduces the amplitudes of the oscillations in the case of a large exchange interaction and a large impurity density. This indicates that in the sense of structural averaging, the period of spin precession is determined mainly by the SOC, and the scattering of magnetic impurities only reduces the amplitudes of precession.

In Figure 5, we present the spatial distribution of $\langle s_z \rangle$ of tunneling electrons in the square lattice. The upper panel corresponds to the case of positive SHC, while the

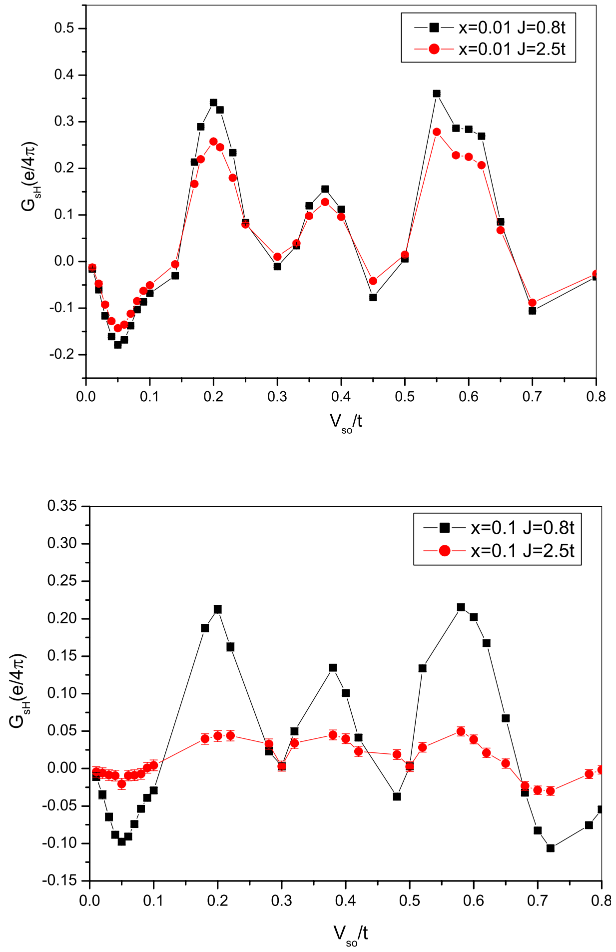

Fig. 4. Spin Hall conductance as a function of the Rashba spin-orbit coupling strength for different values of the impurity density and exchange interaction. The size of system is 20×20, and the electron Fermi energy is $E = -2t$.

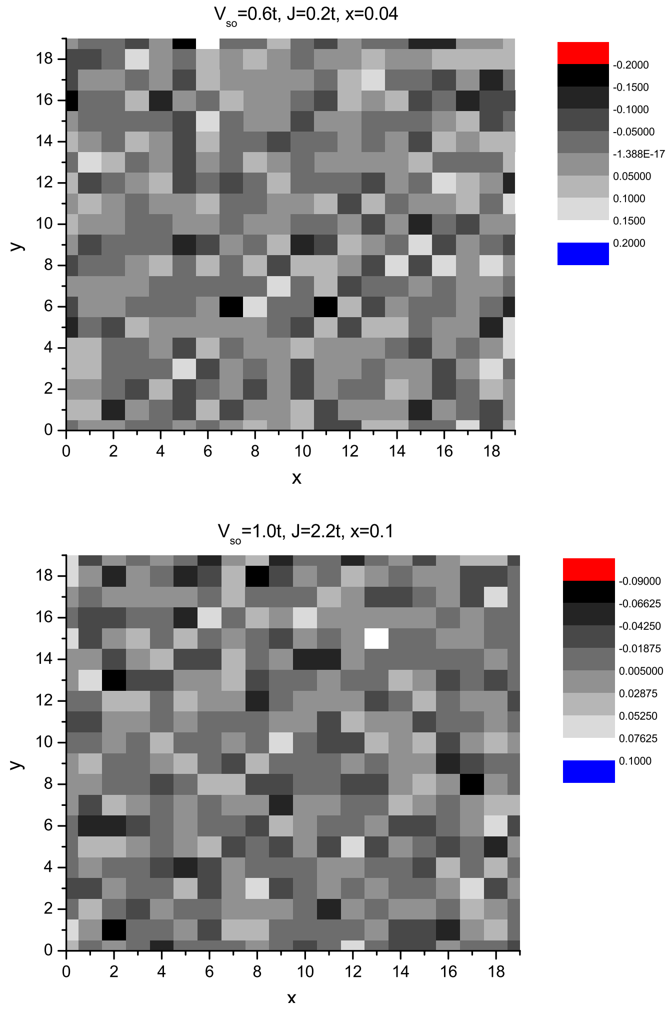

Fig. 5. The Spatial distribution of spin polarization $\langle s_z \rangle$ of tunneling electrons in a square lattice with size $L = 20$.

lower panel shows the case of negative SHC. As can be seen from the figure, the spatial variation of spin polarization in a specific sample due to the spin precession and spin flip-flop scattering is rather random, although a finite G_{sH} can be obtained by structural averaging.

4 Conclusions

In this paper, we have focussed on the study of the spin Hall effect influenced by the scattering of magnetic impurities in systems with Rashba spin-orbit coupling. We find a non-vanishing spin Hall conductance, even in cases where the impurity density and exchange interaction is large. In addition, the change of sign of the spin Hall conductance can also be obtained by varying the impurity density and the strength of the exchange interaction, which is different from that in nonmagnetic impurities doping systems. These results are due to the spin flip-flop scattering caused by magnetic impurities. Such scattering is “biased” in the presence of SOC, leading to an asymmetric effect on the spin up and down states.

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